Kenmore-Town of Tonawanda UFSD

We educate, prepare, and inspire all students to achieve their highest potential

Grade 3 Module 1
Parent Handbook

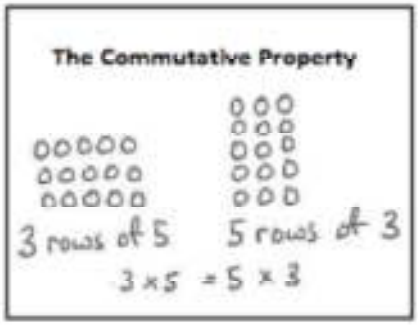
The materials contained within this packet have been taken from the Great Minds curriculum Eureka Math.

Eureka Math™ Tips for Parents

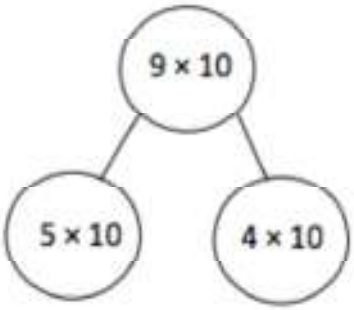
Grade 3 Module 1

Properties of Multiplication and Division and Solving Problems with 2-5 and 10

In this first module of Grade 3, we build on second grade knowledge of addition and work toward greater fluency. We will also be building arrays (arrangements of a set of objects organized into equal groups in rows and columns), and setting the stage for multiplication and division.



An illustration of the Commutative Property



A number bond illustration of the Distributive Property:

9 x 10 = (5 x 10) + (4 x 10)

What Comes After this Module:

In Module 2, students will have opportunities to use tools that build both measurement skills as well as conceptual understanding of metric and time units. Through practical application of measurement skills, students will practice both estimating and rounding numbers.

Terms, Phrases, and Strategies in this Module:

Array: a set of numbers or objects that follow a specific pattern, a matrix

Commutative Property: e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places

Equal groups: with reference to multiplication and division; one factor is the number of objects in a group, and the other is a multiplier that indicates the number of groups

Equation: a statement that 2 expressions are equal, e.g., 3 + 4 = 12

Distributive Property: e.g., 12 × 3 = (10 × 3) + (2 × 3). The 3 is the multiplier and the 12 is decomposed into 10 and 2

Factors: i.e., numbers that are multiplied to obtain a product

Quotient: the answer when one number is divided by another

+ How you can help at home:

- Have your student set out groups of small objects in arrays (equal groups in rows and columns) and write the accompanying multiplication equation
- Encourage your student to practice multiplication facts for 2s, 3s, 4s, 5s, and 10s until they know them fluently

Key Common Core Standards:

- *Represent and solve problems involving multiplication and division*
  - Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities
- *Understand properties of multiplication and the relationship between multiplication and division*
  - Apply properties of operations as strategies to multiply and divide
  - Understand division as an unknown-factor problem
- *Multiply and divide within 100*
  - Fluently multiply and divide within 100
- *Solve problems involving the four operations, and identify and explain patterns in arithmetic*
  - Solve two-step word problems using the four operations

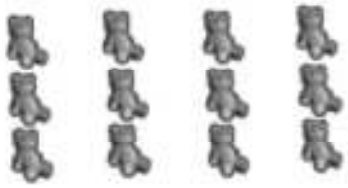
Prepared by Erin Schweng, Math Coach

Welcome to *A Story of Units*!

Each module's parent tip sheet will highlight a new strategy or math model your student will be working on.

Arrays: students worked with arrays toward the end of Grade 2, learning how to use them to show repeated addition. Now, in Grade 3, students put all of their knowledge to work as they learn multiplication and division skills, using arrays to demonstrate the properties of both operations.

(Below) A simple teddy bear array for 3 x 4, or three rows with four in each row



(Left) An array with multiple rows of 3 in each row, showing foundation for multiplication as repeated addition

Read on to learn a little bit about *Eureka Math*, the creators of *A Story of Units:*

Eureka Math is a complete, PreK-12 curriculum and professional development platform. It follows the focus and coherence of the Common Core State Standards (CCSS) and carefully sequences the progression of mathematical ideas into expertly crafted instructional modules.

This curriculum is distinguished not only by its adherence to the CCSS; it is also based on a theory of teaching math that is proven to work. That theory posits that mathematical knowledge is conveyed most effectively when it is taught in a sequence that follows the "story" of mathematics itself. This is why we call the elementary portion of *Eureka Math* "*A Story of Units*." The sequencing has been joined with methods of instruction that have been proven to work, in this nation and abroad. These methods drive student understanding beyond process, to deep mastery of mathematical concepts.

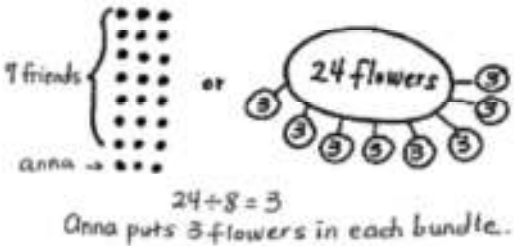
The goal of *Eureka Math* is to produce students who are not merely literate, but fluent, in mathematics. Your student has an exciting year of discovering the story of mathematics ahead!

Sample Problem from Module 1:

(Example taken from Module 1, Lesson 7)

Anna picks 24 flowers.
She makes equal bundles of flowers and gives 1 bundle to each of her 7 friends.
She keeps a bundle for herself too.

How many flowers does Anna put in each bundle?



24 ÷ 8 = 3
Anna puts 3 flowers in each bundle.

Grade 3 • Module 1

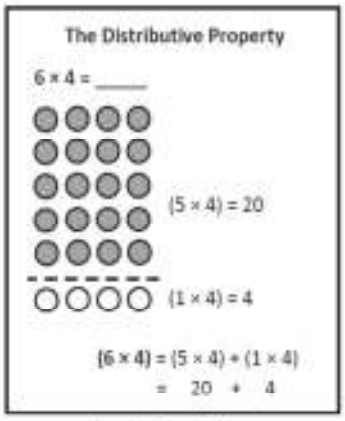
Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

OVERVIEW

This 25-day module begins the year by building on students' fluency with addition and their knowledge of arrays. In Topic A, students initially use repeated addition to find the total from a number of equal groups (**2.OA.4**). As students notice patterns, they let go of longer addition sentences in favor of more efficient multiplication facts (**3.OA.1**). Lessons in Topic A move students' Grade 2 work with arrays and repeated addition a step further by developing skip-counting rows as a strategy for multiplication. Arrays become a cornerstone of the module. Students use the language of multiplication as they understand what factors are and differentiate between the size of groups and the number of groups within a given context. In this module, the factors 2, 3, 4, 5, and 10 provide an entry point for moving into more difficult factors in later modules.

The study of factors links Topics A and B; Topic B extends the study to division. Students understand division as an unknown factor problem and relate the meaning of unknown factors to either the number or the size of groups (**3.OA.2, 3.OA.6**). By the end of Topic B, students are aware of a fundamental connection between multiplication and division that lays the foundation for the rest of the module.

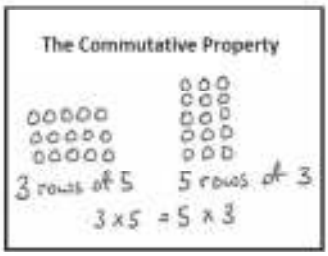
In Topic C, students use the array model and familiar skip-counting strategies to solidify their understanding of multiplication and practice related facts of 2 and 3. They become fluent enough with arithmetic patterns to "add" or "subtract" groups from known products to solve more complex multiplication problems (**3.OA.1**). They apply their skills to word problems using drawings and equations with a symbol to find the unknown factor (**3.OA.3**). This culminates in students using arrays to model the distributive property as they decompose units to multiply (**3.OA.5**).



In Topic D, students model, write, and solve partitive and measurement division problems with 2 and 3 (**3.OA.2**). Consistent skip-counting strategies and the continued use of array models are pathways for students to naturally relate multiplication and division. Modeling advances as students use tape diagrams to represent multiplication and division. A final lesson in this topic solidifies a growing understanding of the relationship between operations (**3.OA.7**).

Topic E shifts students from simple understanding to analyzing the relationship between multiplication and division. Practice of both operations is combined—this time using units of 4—and a lesson is explicitly dedicated to modeling the connection between them (**3.OA.7**). Skip-counting, the distributive property, arrays, number bonds, and tape diagrams are tools for both

operations (**3.OA.1, 3.OA.2**). A final lesson invites students to explore their work with arrays and related facts through the lens of the commutative property as it relates to multiplication (**3.OA.5**).



Topic F introduces the factors 5 and 10, familiar from skip-counting in Grade 2. Students apply the multiplication and division strategies they have used to mixed practice with all of the factors included in Module 1 (**3.OA.1, 3.OA.2, 3.OA.3**). Students model relationships between factors, analyzing the arithmetic patterns that emerge to compose and decompose numbers, as they further explore the relationship between multiplication and division (**3.OA.3, 3.OA.5, 3.OA.7**).

In the final lesson of the module, students apply the tools, representations, and concepts they have learned to problem solving with multi-step word problems using all four operations (**3.OA.3, 3.OA.8**). They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.

Suggested strategy for solving word problems

RDW or Read, Draw, Write (an Equation and a Statement)

Mathematicians and teachers suggest a simple process applicable to all grades:

1) Read.
2) Draw and Label.
3) Write an equation.
4) Write a word sentence (statement).

The more students participate in reasoning through problems with a systematic approach, the more they internalize those behaviors and thought processes.

- What do I see?
- Can I draw something?
- What conclusions can I make from my drawing?

Terminology

New or Recently Introduced Terms

- Array[6] (arrangement of objects in rows and columns)
- Commutative property/commutative (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups)
- Distribute (with reference to the distributive property, e.g., in $12 \times 3 = (10 \times 3) + (2 \times 3)$ the 3 is the multiplier for each part of the decomposition)
- Divide/division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., $15 \div 5 = 3$)

- Factors (numbers that are multiplied to obtain a product)
- Multiplication/multiply (an operation showing how many times a number is added to itself, e.g., 5 × 3 =15)
- Number of groups (factor in a multiplication problem that refers to the total equal groups)
- Parentheses (symbols ( ) used around an expression or numbers within an equation)
- Quotient (the answer when one number is divided by another)
- Rotate (turn, used with reference to turning arrays 90 degrees)
- Row/column (in reference to rectangular arrays)
- Size of groups (factor in a multiplication problem that refers to how many in a group)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the missing factor or quantity in multiplication or division)

Familiar Terms and Symbols

- Add 1 unit, subtract 1 unit (add or subtract a single unit of two, ten, etc.)
- Expression (see expanded description in box above)
- Number bond (illustrates part–part–whole relationship, shown at right)
- Ones, twos, threes, etc. (units of one, two, or three)
- Repeated addition (adding equal groups together, e.g., 2 + 2 + 2 + 2)
- Tape diagram (a method for modeling problems)
- Value (how much)

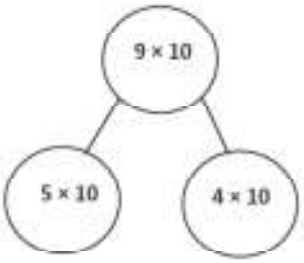
NOTES ON *EXPRESSION, EQUATION, AND NUMBER SENTENCE:*

Please note the descriptions for the following terms, which are frequently misused.

- **Expression:** A number, or any combination of sums, differences, products, or divisions of numbers that evaluates to a number (e.g., 3 + 4, 8 × 3, 15 ÷ 3 as distinct from an equation or number sentence).
- **Equation:** A statement that two expressions are equal (e.g., 3 × ___ = 12, 5 × *b* = 20, 3 + 2 = 5).
- **Number sentence** (also addition, subtraction, multiplication, or division sentence): An equation or inequality for which both expressions are numerical and can be evaluated to a single number (e.g., 4 + 3 = 6 + 1, 2 = 2, 21 > 7 × 2, 5 ÷ 5 =1). Number sentences are either true or false (e.g., 4 + 4 < 6 × 2 and 21 ÷ 7 = 4) and contain no unknowns.

Suggested Tools and Representations

- 18 counters per student
- Tape diagram (a method for modeling problems)
- Number bond (shown at right)
- Array (arrangement of objects in rows and columns)



RDW or Read, Draw, Write (an Equation and a Statement)

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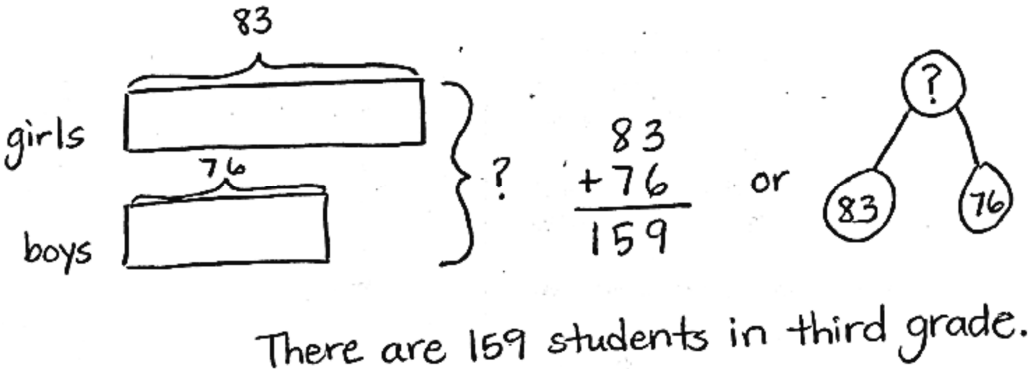
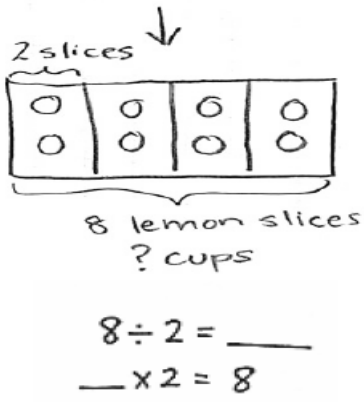
Tape Diagram

A tape diagram is a way for students to visually represent a mathematical problem. It helps students to break down and make sense of a word problem. It provides students access to selecting the appropriate operation as they visualize the relationships between the quantities. Tape diagrams enable students to solve problems efficiently. In Grade 3 Module 1 students will utilize tape diagrams to solve word problems. Several examples follow:

There are 83 girls and 76 boys in third grade. How many total students are there?



Rosie puts 2 lemon slices in each cup of iced tea. She uses a total of 8 slices. How many cups of iced tea does Rosie make?



Rosie make 4 cups of iced tea.

Watch a short video example of a student using a tape diagram at:
https://www.youtube.com/watch?v=GT4fEhfE_8E

Additional Resources for Parents can be found at:

http://greatminds.net/parents

Grade 3 Module 1 Topic A

Multiplication and the Meaning of Factors

Focus Standards:

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

Instructional Days Recommended: 3

Lesson 1 introduces students to multiplication, starting with the concept of repeated addition, which is familiar from Grade 2. Students use repeated addition to find totals; for example, they use counters to make 6 equal groups of 2. They learn to recognize equal groups of counters as units and count units using the language of groups and unit form: "6 equal groups of 2 counters make 12 counters," or "6 twos make 12." By the end of Lesson 1, students use the multiplication symbol to represent these descriptions as more efficient multiplication equations.

In Lesson 2, students relate the equal groups of objects in scattered configurations from Lesson 1 to the array model, exploring the correspondence between 1 equal group and 1 row. They begin to distinguish between the number of groups and the size of groups as they count rows and *how many in 1 row* to write multiplication facts. Students recognize the efficiency of arrays as they skip-count to find totals.

In Lesson 2, students use the following vocabulary: *row, array, number of groups,* and *size of groups.*

Lesson 3 solidifies students' ability to differentiate the meaning of factors. Students model dividing a whole into equal groups as well as analyze equal groups in scattered configurations and arrays to determine whether factors represent the number of groups or the size of groups. They create pictures, number bonds, and multiplication equations to model their understanding.

In this topic, students use a variety of factors since these lessons emphasize understanding the concept of multiplying rather than finding totals. Later topics limit facts to those involving one or two specific factors, allowing students to build fluency with simpler facts before moving on to more difficult ones.

Lesson 1

Objective: Understand *equal groups of* as multiplication.

Key

Homework

1. a. 20; 20; 20
   b. 20; 20; 20
   c. 18; 3, 18; 6, 18
   d. 3, 3, 3, 3, 3, 18; 3, 18; 3, 18
2. Yes; explanations will vary.
3. Picture showing $4 \times 2 = 8$ drawn
4. Pencils circled to show 3 groups of 6; $6 + 6 + 6 = 18$; $3 \times 6 = 18$

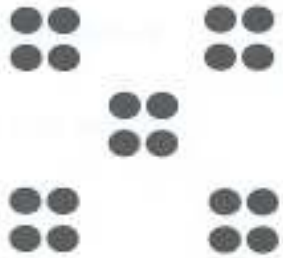
Lesson 1 (continued)

1. Fill in the blanks to make true statements.

a. 4 groups of five = 20

4 fives = 20

4 × 5 = 20



b. 5 groups of four = 20

5 fours = 20

5 × 4 = 20

c. 6 + 6 + 6 = 18

3 groups of six = 18

3 × 6 = 18

d. 3 + 3 + 3 + 3 + 3 + 3 = 18

6 groups of 3 = 18

6 × 3 = 18

Lesson 2

Objective: Relate multiplication to the array model.

Key

Homework

1. a. 3
   b. 2
2. a. 4
   b. 3
3. a. 15
   b. 5×3
4. a. 4
   b. 6×4
8. a. 3 rows of 4 drawn
   b. Answers will vary.
9. 5 rows of 4 drawn; $5 \times 4 = 20$
10. Answers will vary.

Sample Work

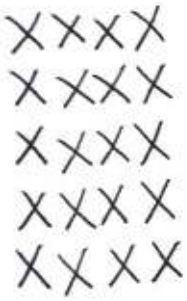
Use the arrays below to answer each set of questions.

1.

a. How many rows of erasers are there? 3

b. How many erasers are there in each row? 2

6. Roger has a collection of stamps. He arranges the stamps into 5 rows of four. Draw an array to represent Roger's stamps. Then, write a multiplication equation to describe the array.



5x4=20

Lesson 3

Objective: Interpret the meaning of factors—the size of the group or the number of groups.

Key

Homework

1. a. 5; 5
   b. 25
   c. 25
2. 4
   a. 6; 4
   b. 4, 24
   c. 24
3. 4
   a. 4; 4
   b. 4, 16
   c. 16
4. 3
   a. 6; 3
   b. 6, 3, 18
   c. 18
5. Array showing 4 rows of 2 or 2 rows of 4 drawn; number bond drawn depending on the array, showing 4 units of 2 equals 8 or 2 units of 4 equals 8

Sample Work

Solve Problems 1–4 using the pictures provided for each problem.

1. There are 5 pineapples in each group. How many pineapples are there in 5 groups?

   a. Number of groups: 5 Size of each group: 5

   b. 5 × 5 = 25

   c. There are 25 pineapples all together.

Grade 3 Module 1 Topic B

Division as an Unknown Factor Problem

Focus Standards:

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Instructional Days Recommended: 3

The study of factors links Topics A and B. Topic B extends the study to division. Students continue to use a variety of factors in this topic as the emphasis in these lessons rests on conceptually understanding division and learning to interpret problems by writing division equations.
Students understand division as an unknown factor problem, and in Lessons 4 and 5, they relate the meaning of the unknown in division to the size of groups and the number of groups, respectively. They work through word problems that help give meaning through context and then analyze more abstract drawings.

In Lesson 6, students explore division in the context of the array model, interpreting arrays by writing division equations. Through the array, students relate the unknown factor in multiplication to the quotient in division. They use arrays to write multiplication equations and find unknown factors, then

write division equations where the quotient represents the same as the unknown factor.

By the end of this topic, students use the vocabulary terms *quotient* and *unknown factor,* and discussion moves toward solidifying understanding of the relationship between multiplication and division.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 4

Objective: Understand the meaning of the unknown as the size of the group in division.

Key

Homework

1. 6
2. 7
3. 5; 5
4. 9, 3; 3; 3
5. 3; 3
6. 4
7. 7; 7
8. Five pencils drawn on each table; 5; 4, 5
9. 4; 20, 5, 4

Sample Work

1. 

12 chairs are divided into 2 equal groups.

There are 6 chairs in each group.

2. 

21 triangles are divided into 3 equal groups.

There are _____ triangles in each group.

3. 

4. 

9. Jenna has markers. The picture shows how she placed them on her desk. Write a division sentence to represent how she equally grouped her markers.

There are 4 markers in each row.

20 ÷ 4 = 5

Lesson 5

Objective: Understand the meaning of the unknown as the number of groups in division.

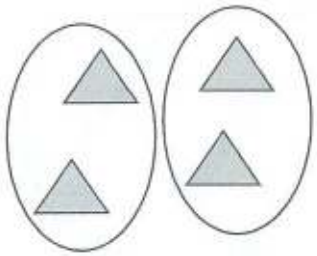
Key

Homework

1. Two groups of 2 shown; 2
2. Three groups of 3 shown; 3; 3
3. Four groups of 3 shown; 4
4. Three groups of 5 shown; 3; 3
5. Two groups of 6 circled
   a. $12 \div 6 = 2$
   b. Number bond showing 2 units of 6 equals 12 drawn
6. a. Count-by fours from 4 to 24 written and drawn
   b. $24 \div 4 = 6$

Sample Work

1.

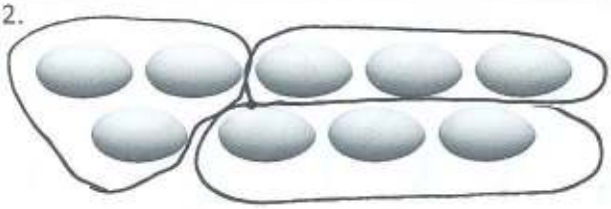


Divide 4 triangles into groups of 2.

There are 2 groups of 2 triangles.

$4 \div 2 = 2$

2.



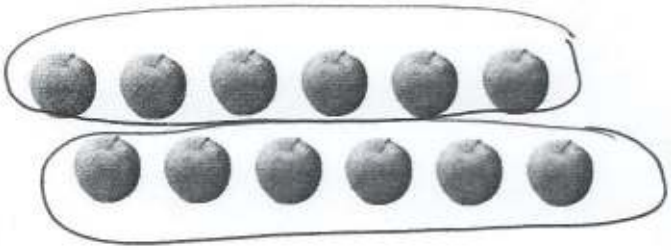
Divide 9 eggs into groups of 3.

There are 3 groups.

$9 \div 3 = 3$

Lesson 5 (continued)

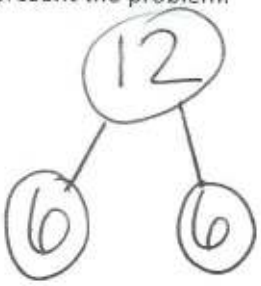
5. Daniel has 12 apples. He puts 6 apples in each bag. Circle the apples to find the number of bags Daniel makes.



a. Write a division sentence where the answer represents the number of Daniel's bags.

$12 \div 6 = 2$

b. Draw a number bond to represent the problem.



Lesson 6

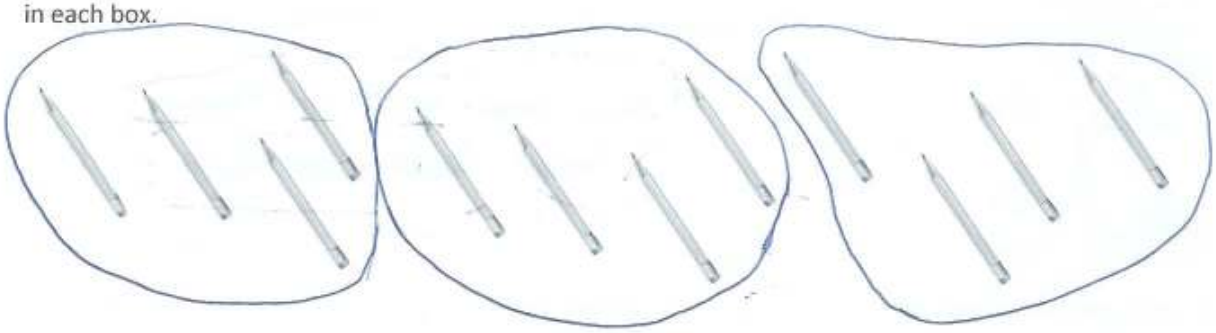
Objective: Interpret the unknown in division using the array model.

Key

1. Three groups of 4 circled; 3; 3; 3
2. Three groups of 4 drawn and circled; 4; 4; 4
3. Array of 3 rows of 4 drawn
    a. 3; 3; the number of groups
    b. 4; 4; the size of each group
4. 6; 6; the size of each group
5. 3; 3; answers will vary
6. Array of 3 rows of 5 drawn

Sample Work

1. Mr. Hannigan puts 12 pencils into boxes. Each box holds 4 pencils. Circle groups of 4 to show the pencils in each box.

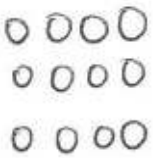


Mr. Hannigan needs 3 boxes.

3 × 4 = 12

12 ÷ 4 = 3

3. Use an array to model Problem 1.



a. 3 × 4 = 12

12 ÷ 4 = 3

The number in the blanks represents the number of groups.

b. 3 × 4 = 12

12 ÷ 3 = 4

The number in the blanks represents the size of each group

5. Nate solves the equation _____ × 5 = 15 by writing and solving 15 ÷ 5 = ____. Explain why Nate's method works.

The unknown factor in _ x 5 = 15 is the same number as the quotient for 15 ÷ 5 = _ .

Grade 3 Module 1 Topic C

Multiplication Using Units of 2 & 3

Focus Standards:

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

3.OA.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

Instructional Days Recommended: 4

In Topic C, students begin building fluency with facts of 2 and 3 using the array model and familiar skip counting strategies.
Lessons 7 and 8 introduce the new complexity of manipulating arrays to study the commutative property. Students learn to distinguish rows from columns as they rotate arrays 90 degrees, noticing that the meaning of the factors changes depending on the orientation of the array. Students write two different multiplication sentences to interpret the same array. These lessons emphasize the equivalence of facts by demonstrating, for example, that 2 groups of 8 and 8 groups of 2 have the same product. Students observe the pattern and begin to recognize commutativity as a strategy for solving twice as many facts.
Lessons 9 and 10 introduce the distributive property as a strategy for multiplication. In Lesson 9, students use arrays to decompose unknown facts

as the sum or difference of two known facts. For example, they analyze an array to see that 7 × 3 can be decomposed as 2 rows of 3 + 5 rows of 3. In Lesson 10, students learn to write the decomposition as (5 × 3) + (2 × 3) = 21. They explain each step of the solving process in anticipation of the work they are expected to complete independently on the Mid-Module Assessment.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 7

Objective: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models.

Key

1. a. Array of 7 rows of 2 drawn

   b. 7, 2, 14

2. a. Array of 2 rows of 7 drawn

   b. 2, 7, 14

3. a. Same array in Problem 1 turned on its side in Problem 2

   b. The meaning of the factors switched; 2 represents size of each group, and 7 represents number of groups in Problem 1; 2 represents number of groups, and 7 represents size of each group in Problem 2

4. a. Answer provided.

   b. $3 \times 2 = 6$

   c. $2 \times 3 = 6$

   d. $2 \times 4 = 8$

   e. $4 \times 2 = 8$

   f. $5 \times 2 = 10$

   g. $2 \times 5 = 10$

   h. $6 \times 2 = 12$

   i. $2 \times 6 = 12$

5. $6 \times 2 = 12$; $2 \times 6 = 12$

6. Agree; array of 2 rows of 8 and array of 8 rows of 2 drawn

7. 2; 7; 2; 10

8. a. Array of 2 rows of 7 drawn

   b. $2 \times 7 = 14$

   c. $7 \times 2 = 14$

Sample Work

1. a. Draw an array that shows 7 rows of 2.

b. Write a multiplication sentence where the first factor represents the number of rows.

7 × 2 = 14

8. Tamia buys 2 bags of candy. Each bag has 7 p dy in it.

a. Draw an array to show how many pieces nia has altogether.

b. Write and solve an equation to describe t

2 x 7 = 14

c. Use the commutative property to write an fferent equation for the array.

7 x 2 = 14

Lesson 8

Objective: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models.

Key

1. Array of 6 rows of 3 drawn
2. Array of 3 rows of 6 drawn
3. 6; 3; 3; 6
4. a. Answer provided
   b. 3 × 5 = 15
   c. 6 × 3 = 18
   d. 3 × 6 = 18
   e. 7 × 3 = 21
   f. 3 × 7 = 21
   g. 8 × 3 = 24
   h. 3 × 9 = 27
   i. 10 × 3 = 30
5. a. 18, matched with Part (e), 18
   b. 15, matched with Part (f), 3
   c. 27, matched with Part (d), 27
6. a. Array of 8 rows of 3 circles drawn
   b. 8 × 3 = 24. Fernando uses 24 pictures.
   c. 2 rows of 3 x's added to array in Part (a)
   d. 10 × 3 = 30
7. a. 4, 3 cents, 12
   b. 7, 3 cents, 21

Sample Work

1. Draw an array that shows 6 rows of 3.

7. Ivania recycles. She gets 3 cents for every can she recycles.
   a. How much money does Ivania make if she recycles 4 cans?

3 × 4 = 12 cents

   b. How much money does Ivania make if she recycles 7 cans?

3 × 7 = 21 cents

Lesson 9

Objective: Find related multiplication facts by adding and subtracting equal groups in array models.

Key

Homework

1. a. 20
   b. 2, 5
   c. 5, 20
2. 14; 12; 2; 14; 7
3. 27; 30; 3; 3; 9
4. a. Array of 5 rows of 4 x's drawn
   b. 20
5. 2 rows of 4 circles added to array in Problem 4
   a. 2, 8
   b. 20, 8
   c. 7

Sample Work

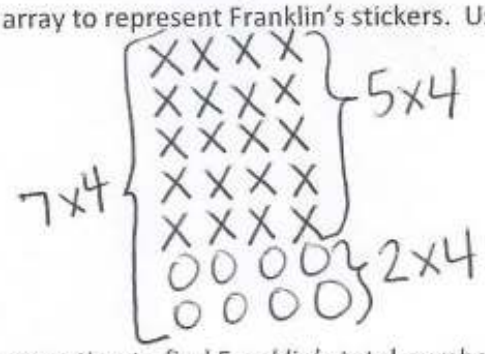
1. Dan organizes his stickers into 3 rows of four. Irene adds 2 more rows of stickers. Complete the equations to describe the total number of stickers in the array.

a. (4 + 4 + 4) + (4 + 4) = 20 (12, 8)

b. 3 fours + 2 fours = 5 fours

c. 5 × 4 = 20

4. Franklin collects stickers. He organizes his stickers in 5 rows of four.
   a. Draw an array to represent Franklin's stickers. Use an x to show each sticker.



   b. Solve the equation to find Franklin's total number of stickers. $5 \times 4 = \underline{20}$

5. Franklin adds 2 more rows. Use circles to show his new stickers on the array in Problem 4(a).
   a. Write and solve an equation to represent the circles you added to the array.

$\underline{2} \times 4 = \underline{8}$

   b. Complete the equation to show how you add the totals of 2 multiplication facts to find Franklin's total number of stickers.

$\underline{20} + \underline{8} = 28$

   c. Complete the unknown to show Franklin's total number of stickers.

$\underline{7} \times 4 = 28$

Lesson 10

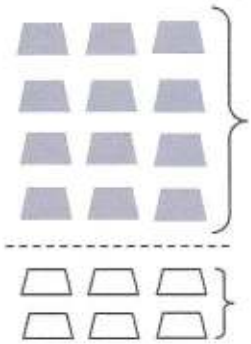
Objective: Model the distributive property with arrays to decompose units as a strategy to multiply.

Homework

1. 18; 6; 6, 18; 18
2. 16; 4, 8; 4, 8; 8, 8; 8, 16
3. a. Array of 5 rows of 3 shown on top shelf, 5; array of 1 row of 3 shown on bottom shelf, 1
   b. 6 × 3 broken into two smaller facts: 5 × 3 = 15 and 1 × 3 = 3; answers of two smaller facts added: 15 + 3; 6 × 3 = 15 + 3 = 18

Sample Work

1. 6 × 3 = 18



(4 × 3) = 12

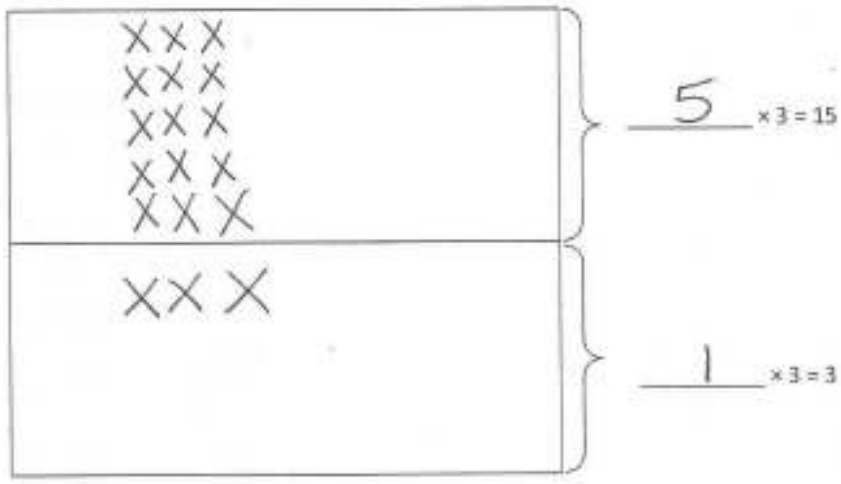
(2 × 3) = 6

12 + 6 = 18

6 × 3 = 18

3. Adriana organizes her books on shelves. She puts 3 books in each row.

a. Fill in the equations on the right. Use them to draw arrays that show the books on Adriana's top and bottom shelves.



b. Adriana calculates the total number of books as shown below. Use the array you drew to help explain Adriana's calculation.

$6 \times 3 = 15 + 3 = 18$

Adriana used $(5 \times 3) + (1 \times 3)$ to find the answer to 6×3. Both ways equal six groups of 3.

Grade 3 Module 1 Topic D

Division Using Units of 2 and 3

Focus Standards:

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.
3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Instructional Days Recommended: 3

In Topic D, students solve two types of division situations—partitive (group size unknown) and measurement (number of groups unknown)—using factors of 2 and 3. Students build on their background knowledge of tape diagrams and apply it to represent division.
In Lesson 11, the tape diagram is used as a tool to help students recognize and distinguish between types of division. By the end of Lessons 11 and 12,

students independently draw and label tape diagrams that help them to compare and analyze problems that may use the same division sentence but have quotients representing different things.
Lesson 13 solidifies growing understanding that the unknown can also be found from the related multiplication sentence. Students initially work through word problems using arrays and tape diagrams to practice solving the two types of division, and then transition to problem solving using abstract division and multiplication equations.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 11

Objective: Model division as the unknown factor in multiplication using arrays and tape diagrams.

Key

1. a. Array drawn showing 2 rows of 5; 10, 5
   b. 2 pears drawn in each unit; unit labeled 2 pears; whole labeled 10 pears and/or ? baskets
2. 5; array drawn showing 3 columns of 5; tape diagram drawn showing 3 groups of 5 is 15
3. 8; array drawn showing 2 columns of 8; tape diagram drawn showing 2 groups of 8 is 16
4. 6; array drawn showing 3 columns of 6; tape diagram drawn showing 3 groups of 6 is 18
5. 7

Sample Work

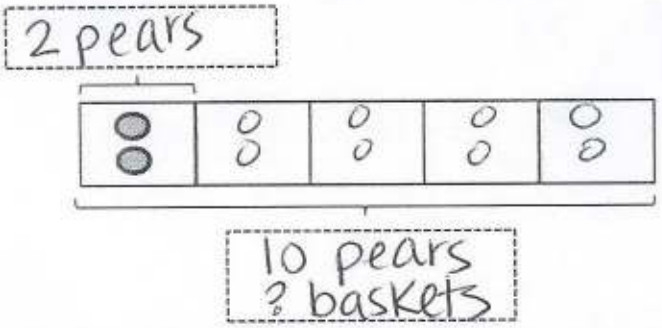
1. Fred has 10 pears. He puts 2 pears in each basket. How many baskets does he have?

   a. Draw an array where each column represents the number of pears in each basket.

Fred put 2 pears in 5 baskets.

10 ÷ 2 = 5

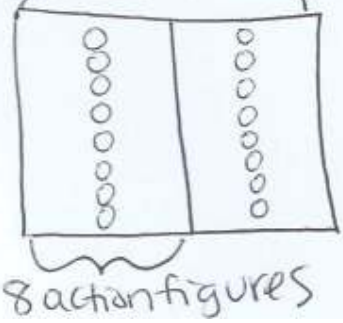
   b. Redraw the pears in each basket as a unit in the tape diagram. Label the diagram with known and unknown information from the problem.



3. Sixteen action figures are arranged equally on 2 shelves. How many action figures are on each shelf? Model the problem with both an array and a labeled tape diagram. Show each column as the number of action figures on each shelf.

Array

Tape Diagram: 16 action figures, 2 shelves



8 action figures

There are 8 action figures on each shelf.
16 ÷ 2 = 8.

Lesson 12

Objective: Interpret the quotient as the number of groups or the number of objects in each group using units of 2.

Key

1. 5 groups of 2 people circled; 5; 5

2. 2 frogs drawn in each group;

   labels will vary; 2; 2; 2

3. First frog matched to 5

   Second frog matched to 8

   Third frog matched to 9

   Fourth frog matched to 7

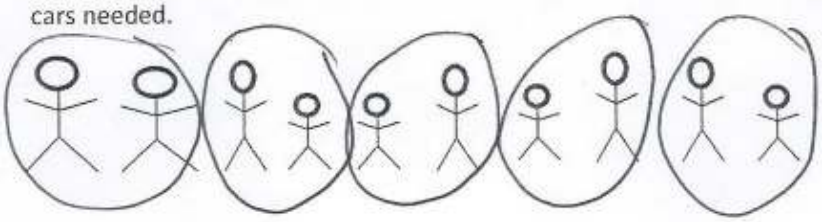
4. 8; labels will vary.

5. 7

6. $8

Sample Work

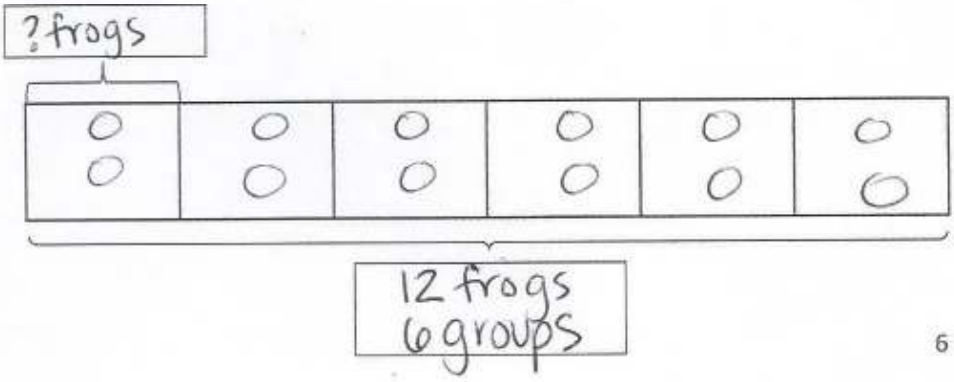
1. Ten people wait in line for the roller coaster. Two people sit in each car. Circle to find the total number of cars needed.



$10 \div 2 =$ 5

There are 5 cars needed.

2. Mr. Ramirez divides 12 frogs equally into 6 groups for students to study. Draw frogs to find the number in each group. Label known and unknown information on the tape diagram to help you solve.



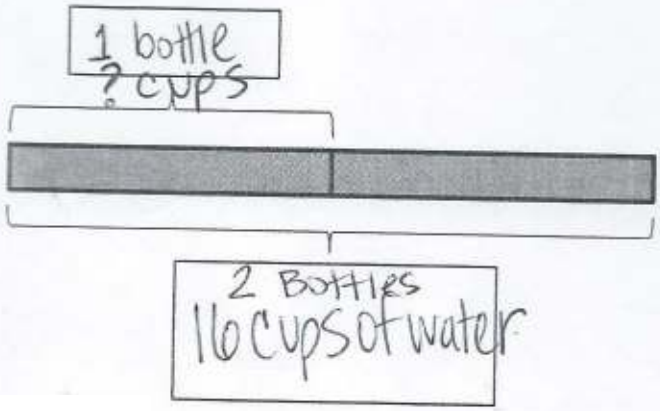
$6 \times$ 2 $= 12$

$12 \div 6 =$ 2

There are 2 frogs in each group.

Lesson 12 (continued)

4. Betsy pours 16 cups of water to equally fill 2 bottles. How many cups of water are in each bottle? Label the tape diagram to represent the problem, including the unknown.



16 ÷ 2 = 8

There are 8 cups of water in each bottle.

Lesson 13

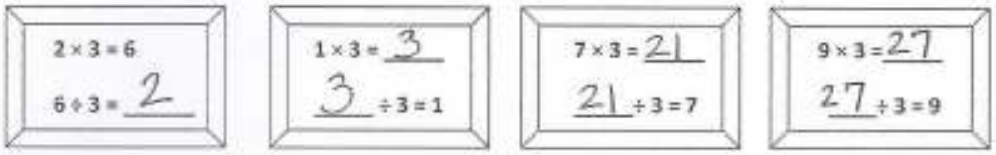
Objective: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

Key

1. 2; 3, 3; 21, 21; 27, 27
2. a. 5 groups of 3 circled; skip-count written as 3, 6, 9, 12, 15
   b. Tape diagram drawn and labeled to represent problem; 15, 5; 5
3. 6
4. 8
5. 9

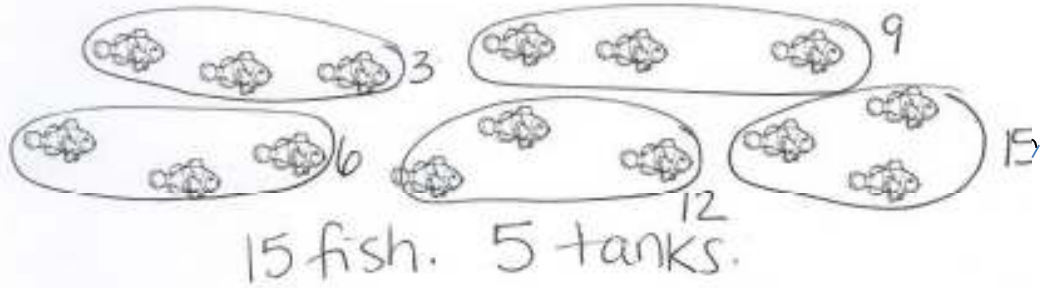
Sample Work

1. Fill in the blanks to make true number sentences.

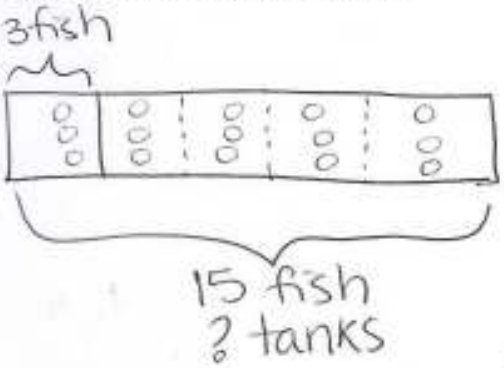


2. Ms. Gillette's pet fish are shown below. She keeps 3 fish in each tank.

   a. Circle to show how many fish tanks she has. Then, skip-count to find the total number of fish.



   b. Draw and label a tape diagram to represent the problem.

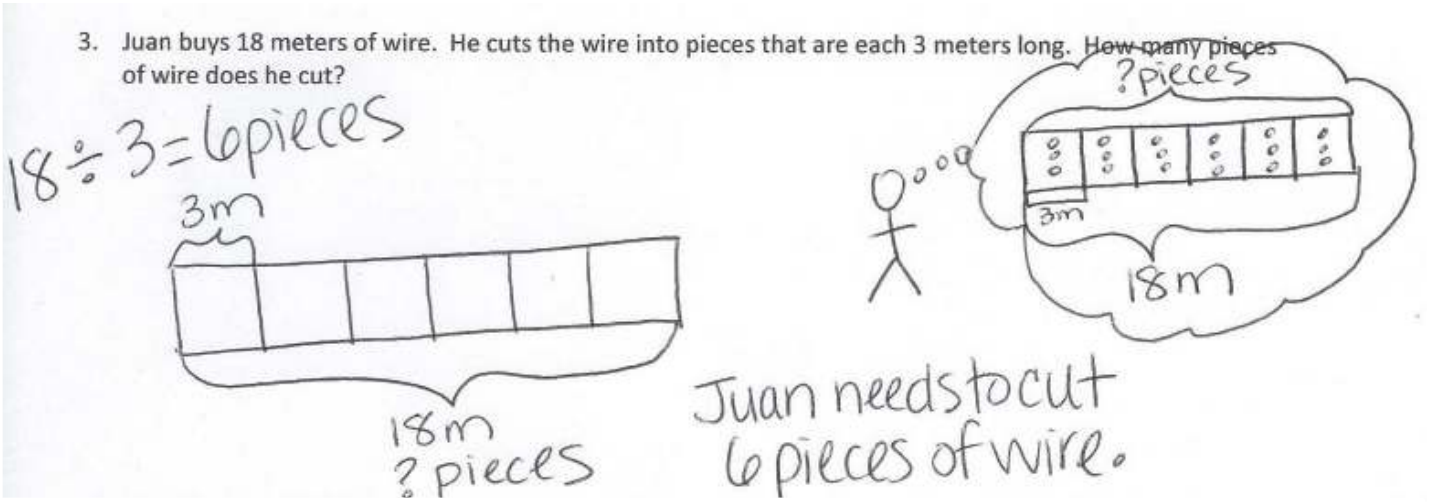


15 ÷ 3 = 5

Ms. Gillette has 5 fish tanks.

Lesson 13 (continued)

3. Juan buys 18 meters of wire. He cuts the wire into pieces that are each 3 meters long. How many pieces of wire does he cut?



Grade 3 Module 1 Topic E

Multiplication and Division Using Units of 4

Focus Standards:

Understand properties of multiplication and the relationship between multiplication and division.
3.OA.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

Multiply and divide within 100.
3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Instructional Days Recommended: 4

Topic E begins by introducing students to multiplication by 4 through skip-counting objects in array models in Lesson 14.
Students revisit the commutative property in Lesson 15, this time modeling commutativity using both arrays and tape diagrams. For example, students might initially draw a 2 × 4 array and a 4 × 2 array. Then, they see 2 bars of equal length; one with 4 equal parts and the other with 2 equal parts. Now,

they have arrays that show (2 × 4) = (4 × 2), as well as tape diagrams that reflect the equality.

In Lesson 16, students examine the distributive property in greater depth. This lesson introduces the 5 + *n* pattern as a strategy for finding unknown facts involving 4. For example, students know that 4 × 5 is 20, so 4 × 6 is the same as 20 + 4 more, which totals 24.

By Lesson 17, practice of multiplication and division facts is dedicated to modeling the relationship between operations using facts of 4.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 14

Objective: Skip-count objects in models to build fluency with multiplication facts using units of 4.

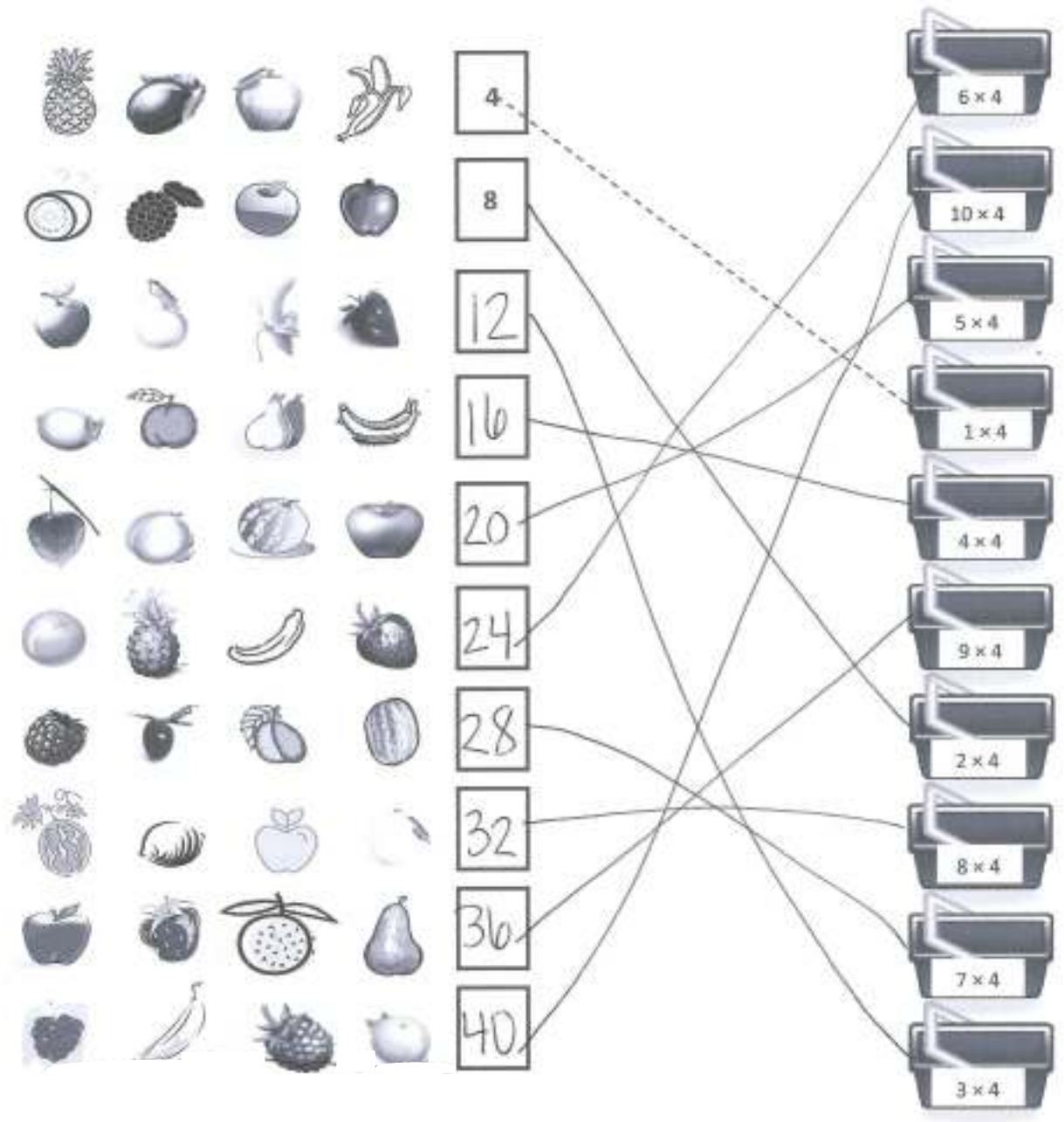
Key

1. 8, 12, 16, 20, 24, 28, 32, 36, 40

   Answer provided; 8 matched to 2 × 4; 12 matched to 3 × 4; 16 matched to 4 × 4; 20 matched to 5 × 4; 24 matched to 6 × 4; 28 matched to 7 × 4; 32 matched to 8 × 4; 36 matched to 9 × 4; 40 matched to 10 × 4
2. Array of 5 rows of 4 drawn; skip-count shown as 4, 8, 12, 16, 20; 20
3. 24; tape diagram drawn and labeled to represent problem
4. 32

Sample Work

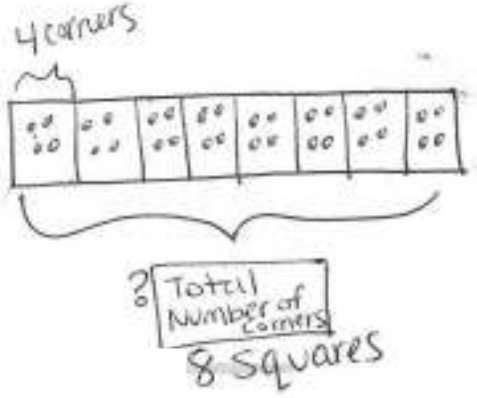
1. Skip-count by fours. Match each answer to the appropriate expression.



2. Lisa places 5 rows of 4 juice boxes in the refrigerator. Draw an array and skip-count to find the total number of juice boxes.

There are 20 juice boxes in total.

4. Find the total number of corners on 8 squares.



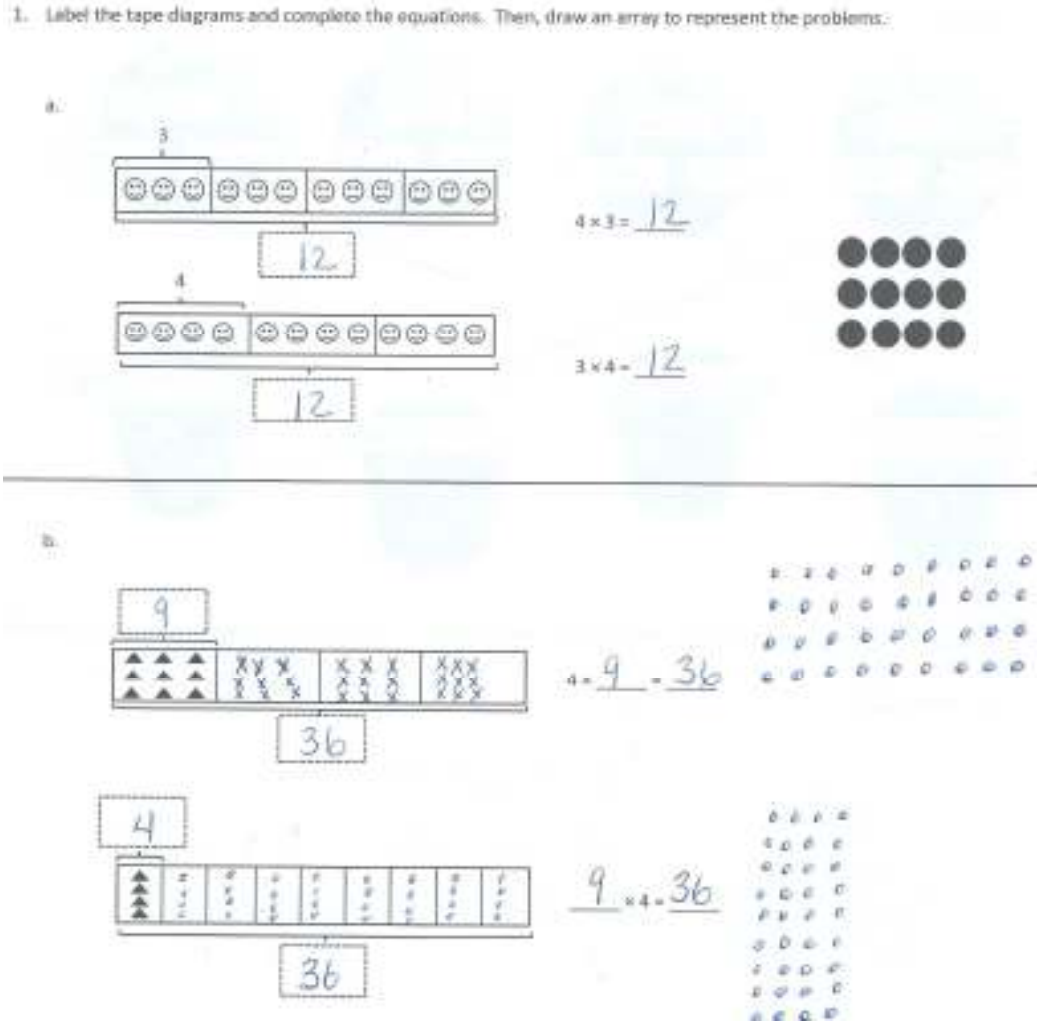
Lesson 15

Objective: Relate arrays to tape diagrams to model the commutative property of multiplication.

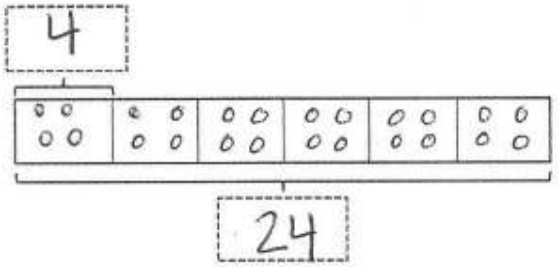
Key

1. a. Top: 12; 12
   Bottom: 12; 12
   b. Top: 9, 36; 9, 36
   Bottom: 4, 36; 9, 36
   Array showing 9 rows of 4 or 4 rows of 9 drawn
   c. Top: 4, 24; 6, 24
   Bottom: 6, 24; 6, 24
   Array showing 6 rows of 4 or 4 rows of 6 drawn
2. Tape diagram drawn and labeled to represent 28 balloons
3. 28

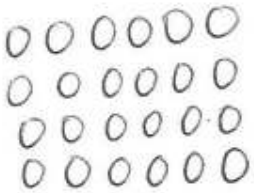
Sample Work

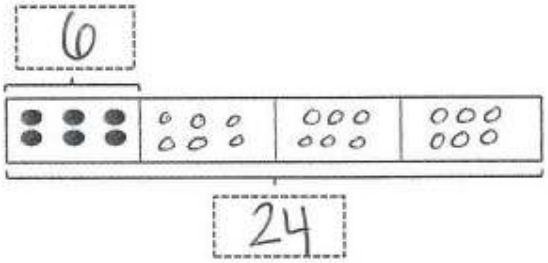


c.



6 × 4 = 24





4 × 6 = 24

Lesson 16

Objective: Use the distributive property as a strategy to find related multiplication facts.

Key

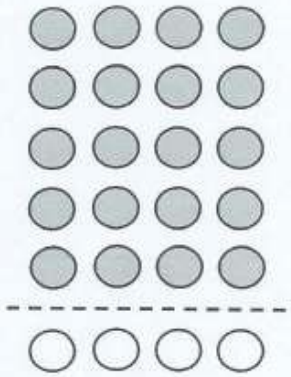
Homework

1. a. 24; 1, 4; 1, 4, 24
   b. 32; 20; 3, 12; 3, 20, 12, 32
2. First sun matched to 24; second sun matched to 28; third sun matched to 32; fourth sun matched to 36
3. 20; 16; 9 fours broken into two smaller facts: 5 fours and 4 fours; sum of two smaller facts found to answer larger fact

Sample Work

1. Label the array. Then, fill in the blanks below to make true number sentences.

a. $6 \times 4 = 24$



$(5 \times 4) = 20$

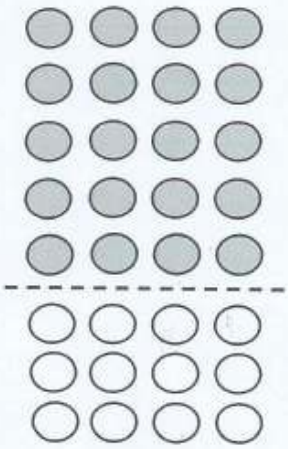
$(1 \times 4) = 4$

$(6 \times 4) = (5 \times 4) + (1 \times 4)$

$= 20 + 4$

$= 24$

b. $8 \times 4 = 32$



$(5 \times 4) = 20$

$(3 \times 4) = 12$

$(8 \times 4) = (5 \times 4) + (3 \times 4)$

$= 20 + 12$

$= 32$

Lesson 17

Objective: Model the relationship between multiplication and division.

Key

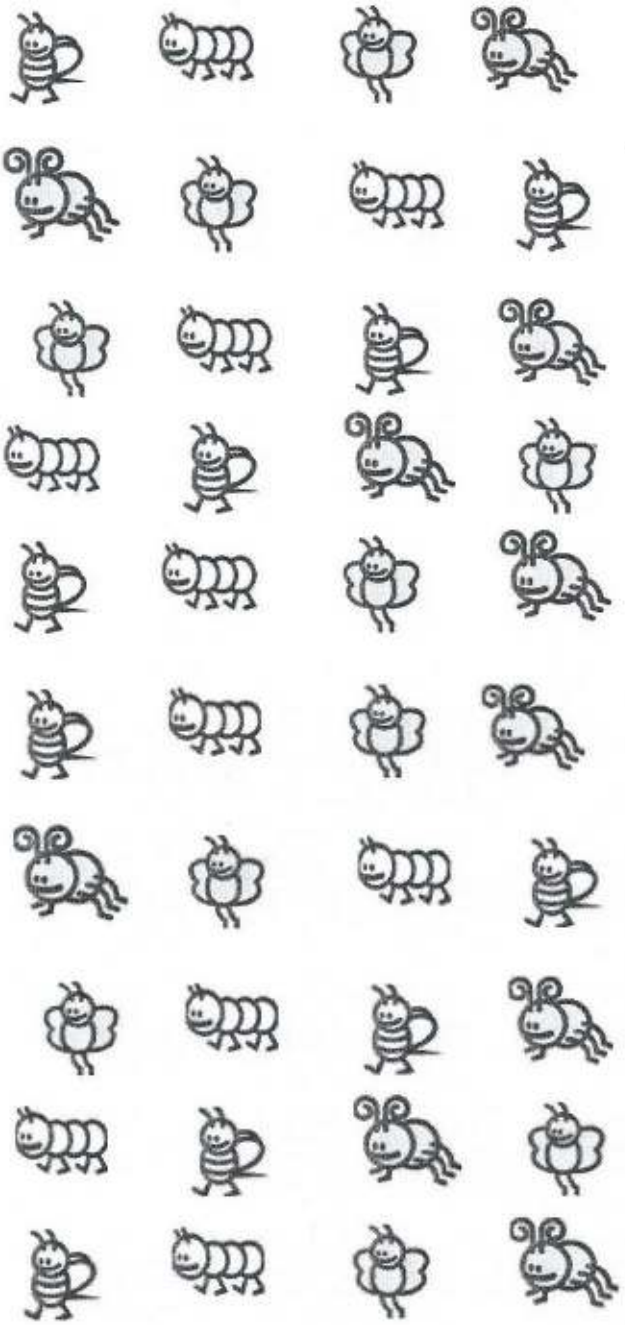
Homework

1. 4; 4
   8; 8
   3; 3
   4; 4
   5, 4; 4, 5
   6, 4; 4, 6
   7, 28; 28, 7
   8, 32; 32, 8
   9, 4, 36; 36, 4, 9
   10, 4, 40; 40, 4, 10
2. 8; tape diagram drawn and labeled to represent the problem
3. 6
4. 12

Lesson 17 (continued)

Sample Work

1. Use the array to complete the related equations.



$1 \times 4 = 4$ $\quad 4 \div 4 = 1$

$2 \times 4 = 8$ $\quad 8 \div 4 = 2$

$3 \times 4 = 12$ $\quad 12 \div 4 = 3$

$4 \times 4 = 16$ $\quad 16 \div 4 = 4$

$5 \times 4 = 20$ $\quad 20 \div 4 = 5$

$6 \times 4 = 24$ $\quad 24 \div 4 = 6$

$7 \times 4 = 28$ $\quad 28 \div 4 = 7$

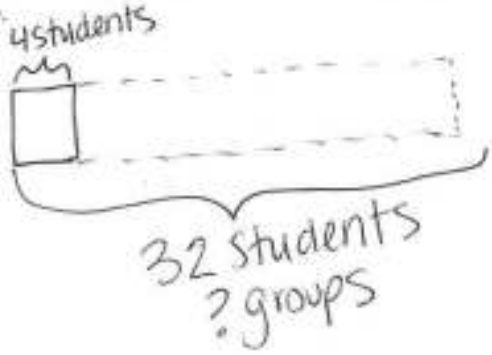
$8 \times 4 = 32$ $\quad 32 \div 4 = 8$

$9 \times 4 = 36$ $\quad 36 \div 4 = 9$

$10 \times 4 = 40$ $\quad 40 \div 4 = 10$

Lesson 17 (continued)

2. The teacher puts 32 students into groups of 4. How many groups does she make? Draw and label a tape diagram to solve.



$32 \div 4 = ?$

$? \times 4 = 32$

8 groups of 4 students equals 32 students.

Grade 3 Module 1 Topic F

Distributive Property and Problem Solving Using Units of 2-5 and 10

Focus Standards:

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Instructional Days Recommended: 4

Topic F introduces the factors 5 and 10, familiar from skip-counting in Grade 2. Students apply the multiplication and division strategies they have learned to mixed practice with all of the factors included in Module 1. Students model relationships between factors and decompose numbers as they further explore the relationship between multiplication and division.

This culminates in Lessons 18 and 19 as students decompose the dividend in a division sentence to practice the distributive property with division. For example, students decompose $28 \div 4$ as $(20 \div 4) + (8 \div 4) = 5 + 2 = 7$. In the final lessons of the module, students apply the tools, representations, and concepts they have learned to solve multi-step word problems. They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.

Lesson 20 focuses on word problems involving multiplication and division, while Lesson 21 increases the complexity of problem solving by including word problems involving all four operations.

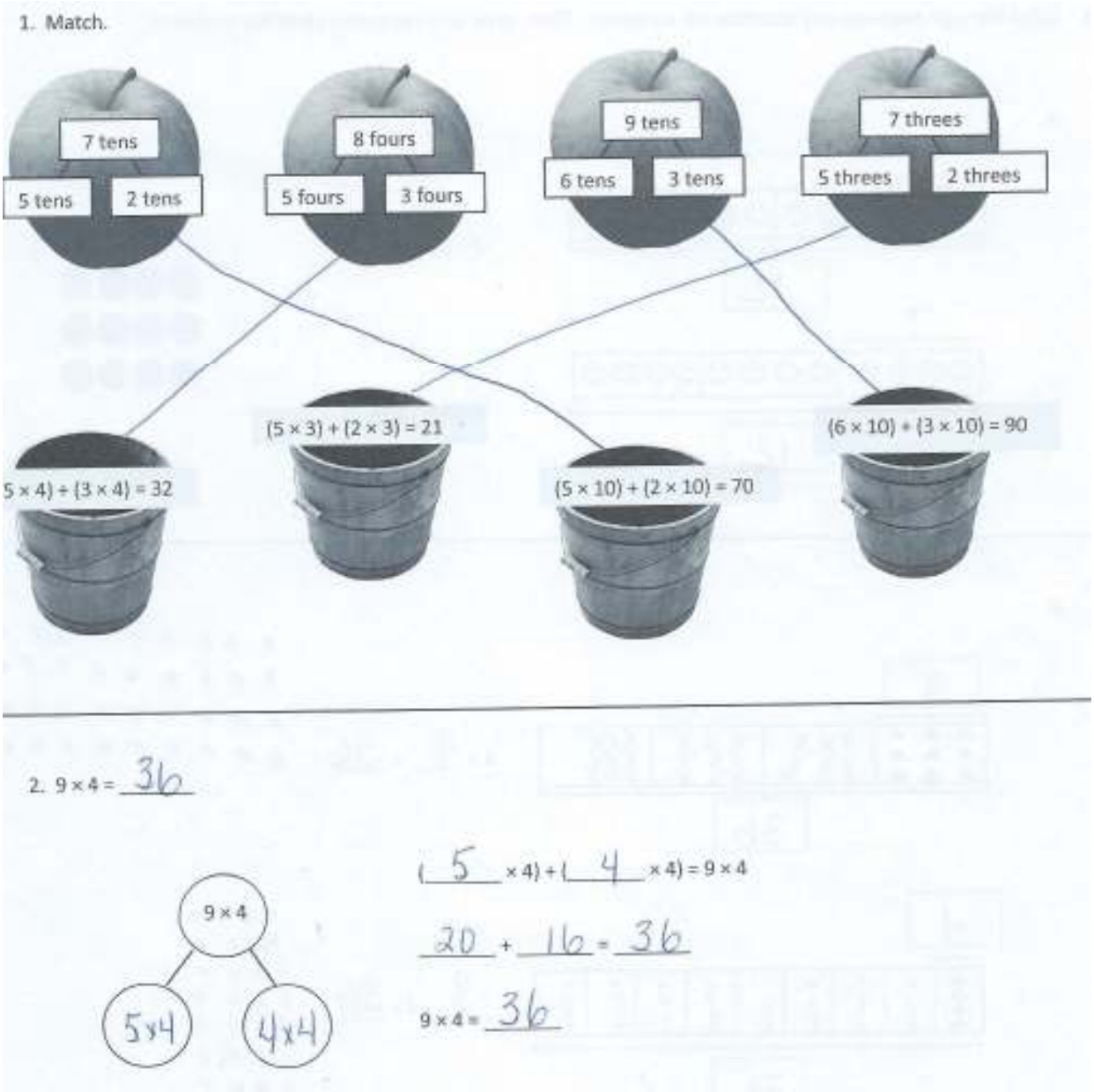
Lesson 18

Objective: Apply the distributive property to decompose units.

Key

1. First apple matched to third bucket; second apple matched to first bucket; third apple matched to fourth bucket; fourth apple matched to second bucket
2. 36; 5 × 4, 4 × 4; 5, 4; 20, 16, 36; 36
3. 40; number bond showing (5 × 4) + (5 × 4) equals 10 × 4 drawn
4. Answers will vary.
5. 70

Sample Work



Lesson 19

Objective: Apply the distributive property to decompose units.

Key

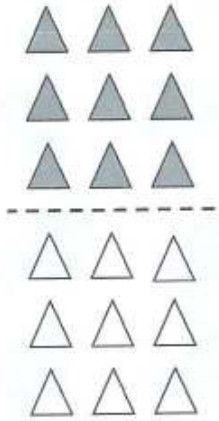
Homework

1. a. 6; 3; 3
   b. 7; 2; 2, 7
   c. 6; 5, 1; 4, 5, 1, 6
   d. 9; 5, 4; 20, 16, 5, 4, 9
2. First white board matched to fourth clipboard; second white board matched to first clipboard; third white board matched to third clipboard; fourth white board matched to second clipboard
3. 35 ÷ 5 broken into two smaller facts: 20 ÷ 5 and 15 ÷ 5; sum of two smaller facts found to answer larger fact

Sample Work

1. Label the array. Then, fill in the blanks to make true number sentences.

a. $18 \div 3 = 6$



$(9 \div 3) = 3$

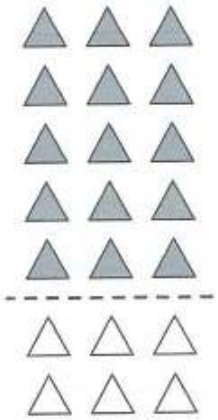
$(9 \div 3) =$ ____

$(18 \div 3) = (9 \div 3) + (9 \div 3)$

$= 3 + 3$

$= 6$

b. $21 \div 3 = 7$



$(15 \div 3) = 5$

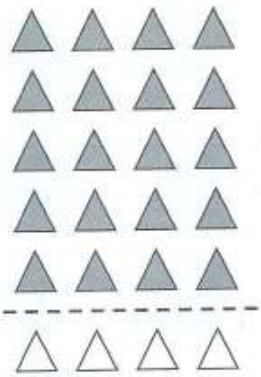
$(6 \div 3) = 2$

$(21 \div 3) = (15 \div 3) + (6 \div 3)$

$= 5 + 2$

$= 7$

c. $24 \div 4 = 6$



$(20 \div 4) = 5$

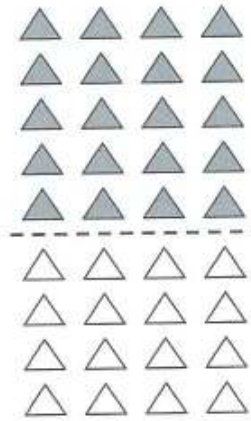
$(4 \div 4) = 1$

$(24 \div 4) = (20 \div 4) + (4 \div 4)$

$= 5 + 1$

$= 6$

d. $36 \div 4 = 9$



$(20 \div 4) = 5$

$(16 \div 4) = 4$

$(36 \div 4) = (20 \div 4) + (16 \div 4)$

$= 5 + 4$

$= 9$

Lesson 20

Objective: Solve two-step word problems involving multiplication and division, and assess the reasonableness of answers.

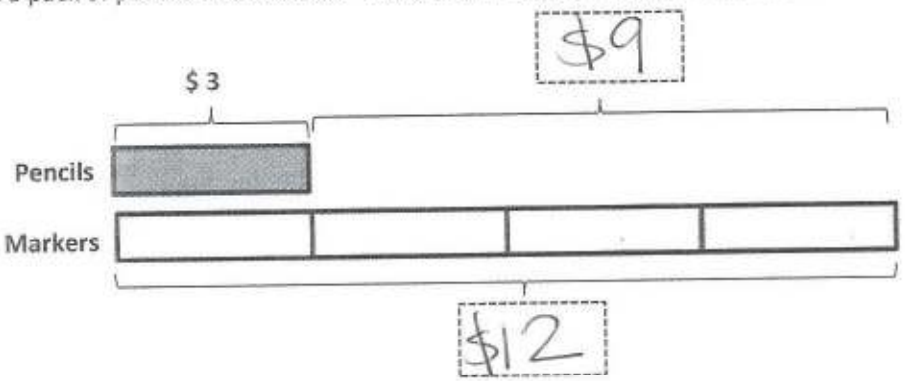
Key

Homework

1. Tape diagram labeled
   a. $12
   b. $9
2. Tape diagram labeled
   a. 6
   b. 24
3. 4 green and 5 purple
4. 9
5. 4

Sample Work

1. Jerry buys a pack of pencils that costs $3. David buys 4 sets of markers. Each set of markers also costs $3.



a. What is the total cost of the markers?

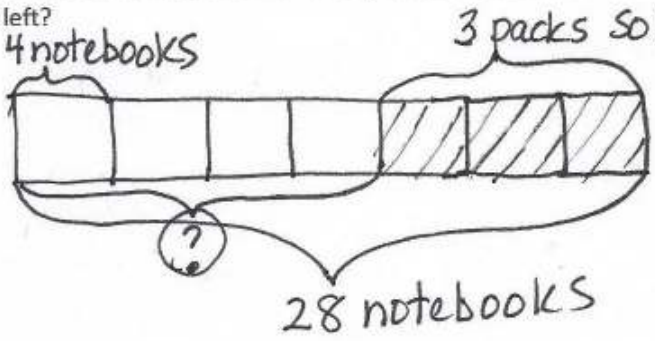
$12 4 x $3 = $12
The markers cost $12.

b. How much more does David spend on 4 sets of markers than Jerry spends on a pack of pencils?

$12 - $3 = $9.
David spends $9 more than Jerry.

Lesson 20 (continued)

5. The store has 28 notebooks in packs of 4. Three packs of notebooks are sold. How many packs of notebooks are left?



28 ÷ 4 = 7
7 packs - 3 packs = 4 pack
There are 4 packs left

Lesson 21

Objective: Solve two-step word problems involving all four operations, and assess the reasonableness of answers.

Key

1. Tape diagram labeled; 4 × 8 = 32; 32 + 5 = 37; 37

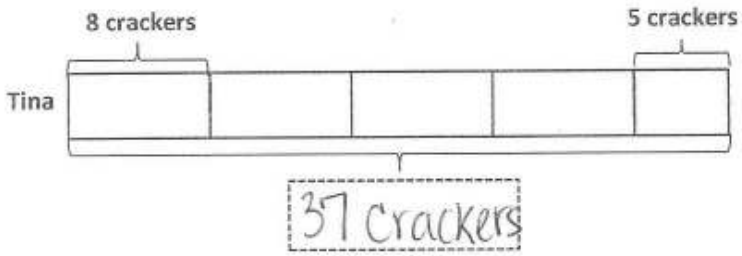
2. Tape diagrams labeled; 23

3. Tape diagram drawn and labeled to represent problem; 12

4. 3

Sample Work

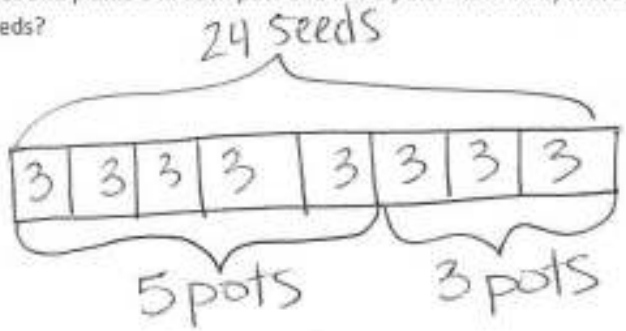
1. Tina eats 8 crackers for a snack each day at school. On Friday, she drops 3 and only eats 5. Write and solve an equation to show the total number of crackers Tina eats during the week.



4 x 8 = 32
32 + 5 = 37

Tina eats 37 crackers.

4. Anna buys 24 seeds and plants 3 in each pot. She has 5 pots. How many more pots does Anna need to plant all of her seeds?



24 ÷ 3 = 8 pots

8 pots - 5 pots = 3 pots

Anna needs 3 more pots to plant all of her seeds.